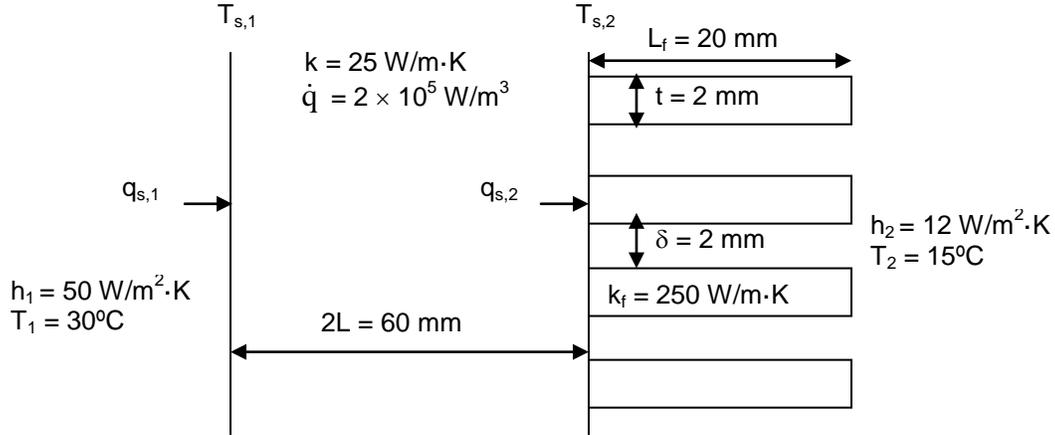


## PROBLEM 07

**KNOWN:** Wall with known heat generation rate, thermal conductivity, and thickness. Dimensions and thermal conductivity of fins. Heat transfer coefficients and environment temperatures.

**FIND:** Maximum temperature.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) Wall surface temperatures are uniform. (3) No contact resistance between fins and wall, (4) Heat transfer from the fin tips can be neglected.

**ANALYSIS:** The temperature distribution in a wall with uniform volumetric heat generation and specified temperature boundary conditions is, from Equation 3.46

$$T(x) = \frac{\dot{q}L^2}{2k} \left( 1 - \frac{x^2}{L^2} \right) + \frac{T_{s,2} - T_{s,1}}{2} \frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \quad (1)$$

The heat transfer rates at the two surfaces, for a wall section of area  $A$ , can be found from Fourier's law:

$$q_{s,1} = -kA \left. \frac{dT}{dx} \right|_{x=-L} = -\dot{q}LA - kA \frac{T_{s,2} - T_{s,1}}{2L} \quad (2)$$

$$q_{s,2} = -kA \left. \frac{dT}{dx} \right|_{x=L} = \dot{q}LA - kA \frac{T_{s,2} - T_{s,1}}{2L} \quad (3)$$

We can express these same heat transfer rates alternatively, as follows:

$$q_{s,1} = h_1 A (T_1 - T_{s,1}) \quad (4)$$

$$q_{s,2} = h_2 A_t (T_{s,2} - T_2) \eta_o \quad (5)$$

where  $\eta_o$  is given by Equation 3.107. Equating the two expressions for  $q_{s,1}$ , Equations (2) and (4), and equating the expressions for  $q_{s,2}$ , Equations (3) and (5), and solving for  $T_{s,1}$  and  $T_{s,2}$  yields

Continued...

**PROBLEM 08 (Cont.)**

$$T_{s,1} = \frac{\left(\frac{k}{2L} + h_2\tilde{A}\right)h_1T_1 + \frac{k}{2L}h_2\tilde{A}T_2 + \left(\frac{k}{L} + h_2\tilde{A}\right)\dot{q}L}{\frac{kh_1}{2L} + h_1h_2\tilde{A} + \frac{kh_2\tilde{A}}{2L}}$$

$$T_{s,2} = \frac{\frac{k}{2L}h_1T_1 + \left(\frac{k}{2L} + h_1\right)h_2\tilde{A}T_2 + \left(\frac{k}{L} + h_1\right)\dot{q}L}{\frac{kh_1}{2L} + h_1h_2\tilde{A} + \frac{kh_2\tilde{A}}{2L}}$$

where

$$\tilde{A} = \frac{A_t\eta_o}{A} = \frac{A_t}{A} - \frac{NA_f}{A}(1 - \eta_f)$$

Performing the calculations:

$$m = \sqrt{\frac{h_2P}{k_f A_c}} = \sqrt{\frac{2h_2}{k_f t}} = \sqrt{\frac{2 \times 12 \text{ W/m}^2 \cdot \text{K}}{250 \text{ W/m} \cdot \text{K} \times 0.002 \text{ m}}} = 6.9 \text{ m}^{-1}$$

$$\eta_f = \frac{\tanh(mL_f)}{mL_f} = \frac{\tanh(6.9 \text{ m}^{-1} \times 0.02 \text{ m})}{6.9 \text{ m}^{-1} \times 0.02 \text{ m}} = 0.994$$

$$\frac{NA_f}{A} = \frac{N2wL_f}{(\delta+t)Nw} = \frac{2L_f}{\delta+t} = \frac{2 \times 0.02 \text{ m}}{0.004 \text{ m}} = 10.0$$

$$\frac{A_t}{A} = \frac{NA_f}{A} + \frac{A_b}{A} = \frac{NA_f}{A} + \frac{\delta Nw}{(\delta+t)Nw} = \frac{NA_f}{A} + \frac{\delta}{\delta+t} = 10. + \frac{0.002 \text{ m}}{0.004 \text{ m}} = 10.5$$

$$\tilde{A} = 10.5 - 10.(1 - 0.994) = 10.4$$

$$h_2\tilde{A} = 12 \text{ W/m}^2 \cdot \text{K} \times 10.4 = 125 \text{ W/m}^2 \cdot \text{K}$$

$$\frac{k}{2L} = \frac{25 \text{ W/m} \cdot \text{K}}{0.06 \text{ m}} = 417 \text{ W/m}^2 \cdot \text{K}$$

Thus

$$T_{s,1} = \frac{\left( \begin{array}{l} (417+125) \text{ W/m}^2 \cdot \text{K} \times 50 \text{ W/m}^2 \cdot \text{K} \times 30^\circ\text{C} \\ + 417 \text{ W/m}^2 \cdot \text{K} \times 125 \text{ W/m}^2 \cdot \text{K} \times 15^\circ\text{C} \\ + (2 \times 417 + 125) \text{ W/m}^2 \cdot \text{K} \times 2 \times 10^5 \text{ W/m}^3 \times 0.03 \text{ m} \end{array} \right)}{\left( \begin{array}{l} (417 \times 50 \\ + 50 \times 125 \\ + 417 \times 125) (\text{W/m}^2 \cdot \text{K})^2 \end{array} \right)}$$

Continued...

### PROBLEM 08 (Cont.)

$$T_{s,1} = 92.7^\circ\text{C}$$

Similarly,

$$T_{s,2} = 85.8^\circ\text{C}$$

The location of the maximum temperature in the wall can be found by setting the gradient of the temperature (from Equation (1)) to zero:

$$\frac{dT}{dx} = -\frac{\dot{q}x}{k} + \frac{T_{s,2} - T_{s,1}}{2L} = 0$$

Thus,  $x_{\max} = k \frac{T_{s,2} - T_{s,1}}{2L\dot{q}}$ . Substituting this back into the temperature distribution,

$$\begin{aligned} T_{\max} &= \frac{\dot{q}L^2}{2k} + \frac{k(T_{s,2} - T_{s,1})^2}{8L^2\dot{q}} + \frac{T_{s,1} + T_{s,2}}{2} \\ &= \frac{2 \times 10^5 \text{ W/m}^3 \times (0.03 \text{ m})^2}{2 \times 25 \text{ W/m} \cdot \text{K}} + \frac{25 \text{ W/m} \cdot \text{K} (85.8^\circ\text{C} - 92.7^\circ\text{C})^2}{8 \times (0.03 \text{ m})^2 \times 2 \times 10^5 \text{ W/m}^3} \\ &\quad + \frac{92.7^\circ\text{C} + 85.8^\circ\text{C}}{2} = 93.7^\circ\text{C} \end{aligned} \quad <$$