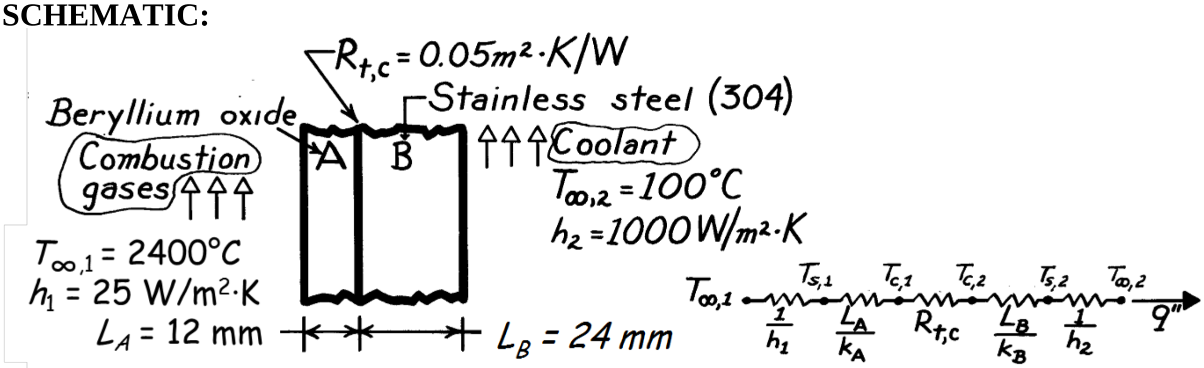
PROBLEM 04

KNOWN: Materials and dimensions of a composite wall separating a combustion gas from a liquid coolant.

FIND: (a) Heat loss per unit area, and (b) Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Constant properties, (4) Negligible radiation effects.

PROPERTIES: *Table A-1*, St. St. (304) $\left(\bar{T} \approx 1000K\right)$: k = 25.4 W/m·K; *Table A-2*, Beryllium Oxide (T ≈ 1500K): k = 21.5 W/m·K.

ANALYSIS: (a) The desired heat flux may be expressed as

$$q'' = \frac{T_{\infty,1} - T_{\infty,2}}{\frac{1}{h_1} + \frac{L_A}{k_A} + R_{t,c} + \frac{L_B}{k_B} + \frac{1}{h_2}} = \frac{(2400 - 100)^\circ C}{\left[\frac{1}{25} + \frac{0.012}{21.5} + 0.05 + \frac{0.024}{25.4} + \frac{1}{1000}\right]\frac{m^2.K}{W}}$$

$$q'' = 24{,}860 \ W/m^2.$$ <

(b) The composite surface temperatures may be obtained by applying appropriate rate equations. From the fact that $q'' = h_1 \left(T_{\infty,1} - T_{s,1}\right)$, it follows that

$$T_{s,1} = T_{\infty,1} - \frac{q''}{h_1} = 2400^\circ C - \frac{24{,}860 \ W/m^2}{25 \ W/m^2 \cdot K} = 1405^\circ C.$$

With $q'' = \left(k_A / L_A\right)\left(T_{s,1} - T_{c,1}\right)$, it also follows that

$$T_{c,1} = T_{s,1} - \frac{L_A q''}{k_A} = 1405^\circ C - \frac{0.012 m \times 24{,}860 \ W/m^2}{21.5 \ W/m \cdot K} = 1392^\circ C.$$

Similarly, with $q'' = \left(T_{c,1} - T_{c,2}\right)/R_{t,c}$

$$T_{c,2} = T_{c,1} - R_{t,c} q'' = 1392^\circ C - 0.05 \frac{m^2 \cdot K}{W} \times 24{,}860 \frac{W}{m^2} = 148^\circ C$$

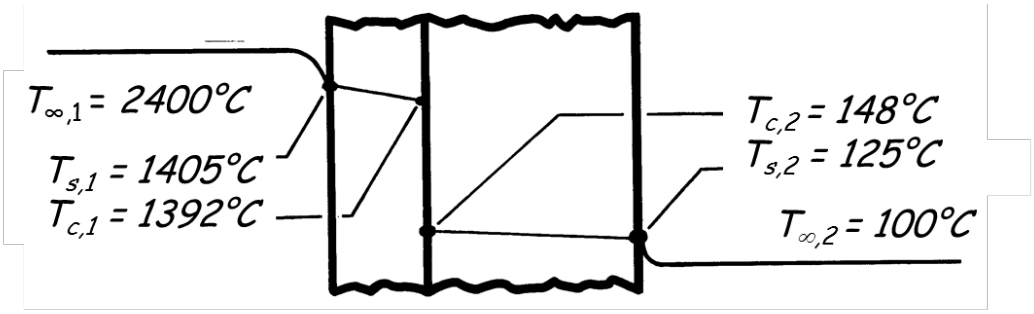
Continued …

PROBLEM 04 (Cont.)

and with $q'' = (k_B / L_B)(T_{c,2} - T_{s,2})$,

$$T_{s,2} = T_{c,2} - \frac{L_B q''}{k_B} = 148^\circ C - \frac{0.024m \times 24{,}860 \text{ W/m}^2}{25.4 \text{ W/m} \cdot K} = 125^\circ C.$$

The temperature distribution is therefore of the following form:



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COMMENTS: (1) The calculations may be checked by recomputing q'' from

$$q'' = h_2 (T_{s,2} - T_{\infty,2}) = 1000 \text{ W/m}^2 \cdot K (125\text{-}100)\ ^\circ C = 25{,}000 \text{ W/m}^2$$

The discrepancy is due to roundoff error in the temperature.

(2) The initial *estimates* of the mean material temperatures are in error, particularly for the stainless steel. For improved accuracy the calculations should be repeated using k values corresponding to T ≈ 1400°C for the oxide and T ≈ 140°C for the steel.

(3) The major contributions to the total resistance are made by the combustion gas boundary layer and the contact, where the temperature drops are largest.