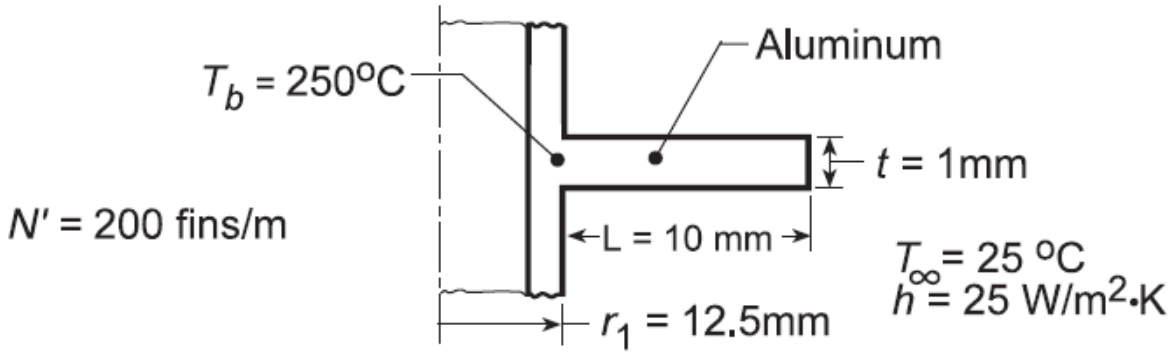
PROBLEM 08

KNOWN: Dimensions and base temperature of an annular, aluminum fin of rectangular profile. Ambient air conditions.

FIND: (a) Fin heat loss, (b) Heat loss per unit length of tube with 200 fins spaced at 5 mm increments.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation and contact resistance, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) The fin parameters for use with Figure 3.20 are

$$r_{2c} = r_2 + t/2 = (12.5\,\text{mm} + 10\,\text{mm}) + 0.5\,\text{mm} = 23\,\text{mm} = 0.023\,\text{m}$$

$$r_{2c}/r_1 = 1.84 \qquad L_c = L + t/2 = 10.5\,\text{mm} = 0.0105\,\text{m}$$

$$A_p = L_c t = 0.0105\,\text{m} \times 0.001\,\text{m} = 1.05\times10^{-5}\,\text{m}^2$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.0105\,\text{m})^{3/2}\left(\frac{25\,\text{W/m}^2\cdot\text{K}}{240\,\text{W/m}\cdot\text{K}\times1.05\times10^{-5}\,\text{m}^2}\right)^{1/2} = 0.15\,.$$

Hence, the fin effectiveness is $\eta_f \approx 0.97$, and from Eq. 3.91 and Fig. 3.6, the fin heat rate is

$$q_f = \eta_f q_{max} = \eta_f h A_{f(ann)} \theta_b = 2\pi\eta_f h\left(r_{2,c}^2 - r_1^2\right)\theta_b$$

$$q_f = 2\pi\times0.97\times25\,\text{W/m}^2\cdot\text{K}\times\left[(0.023\,\text{m})^2 - (0.0125\,\text{m})^2\right]225^\circ\text{C} = 12.8\,\text{W}\,. \qquad <$$

(b) Recognizing that there are N = 200 fins per meter length of the tube, the total heat rate considering contributions due to the fin and base (unfinned) surfaces is

$$q' = N'q_f + h(1 - N't)2\pi r_1\theta_b$$

$$q' = 200\,\text{m}^{-1}\times12.8\,\text{W} + 25\,\text{W/m}^2\cdot\text{K}\left(1 - 200\,\text{m}^{-1}\times0.001\,\text{m}\right)\times2\pi\times(0.0125\,\text{m})225^\circ\text{C}$$

$$q' = (2560\,\text{W} + 353\,\text{W})/\text{m} = 2.91\,\text{kW/m}\,. \qquad <$$

COMMENTS: Note that, while covering only 20% of the tube surface area, the tubes account for more than 85% of the total heat dissipation.